

Math 180, Quiz 7 Retest: Tuesday, March 19, 2013

Take fifteen minutes to complete this quiz. Please show all your work, and write your name on the front and back of the paper before turning it in. Make sure to show all relevant work.

1. (4 pts) Find the absolute extrema of $f(x) = x \ln(x) + 2$ on the interval $[e^{-2}, 1]$.

$$f'(x) = (x)' \ln(x) + (1) \ln(x) + 0 = \ln(x) + 1$$

$$\ln(x) + 1 = 0 \Rightarrow \ln(x) = -1 \Rightarrow x = e^{-1}$$

$$f(e^{-2}) = e^{-2}(-2) + 2 = 2 - \frac{2}{e^2}$$

$$f(e^{-1}) = 2 - \frac{1}{e}$$

min

$$f(1) = 1(0) + 2 = 2$$

max

2. (5 pts) A 5 m-long log rolls down a snow-covered mountain, forming a cylinder of constant height but increasing diameter. It accumulates snow at a rate of $1 \text{ m}^3/\text{s}$. How quickly is the diameter increasing when the snow-covered log is $1/3 \text{ m}$ across? (You probably want to work with radii first and then convert to diameters at the end.)

$$D = 2r$$

$$V = \pi r^2 L$$

$$r = \frac{1}{6} \text{ m}$$

$$\frac{dV}{dt} = 1 \text{ m}^3/\text{s}$$

$$D' = 2r'$$

$$V' = 2\pi r L r'$$

$$\Rightarrow r' = \frac{V'}{2\pi r L} = \frac{1 \text{ m}^3/\text{s}}{2\pi \left(\frac{1}{6} \text{ m}\right) (5 \text{ m})} = \frac{6}{10\pi} \text{ m/s} \Rightarrow D' = \frac{6}{5\pi} \text{ m/s}$$

3. (1 pt) Suppose f and f^{-1} are differentiable functions, and that you are given the values

| x | $f(x)$ | $f'(x)$ |
|-------|--------|---------|
| 0 | $3/2$ | $1/2$ |
| 1 | 2 | $1/2$ |
| $3/2$ | $19/8$ | 1 |
| 2 | 3 | $3/2$ |
| $5/2$ | $31/8$ | 2 |
| 3 | 5 | $5/2$ |

What is $(f^{-1})'(3/2)$? (Show at least enough work that I can tell you didn't just make a lucky guess!)

Recall:

$$f(f^{-1}(x)) = x$$

$$f'(f^{-1}(x)) (f^{-1})'(x) = 1$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$f^{-1}(3/2) = 0$$

$$f'(0) = 1/2$$

$$(f^{-1})'(3/2) = \frac{1}{1/2} = 2$$